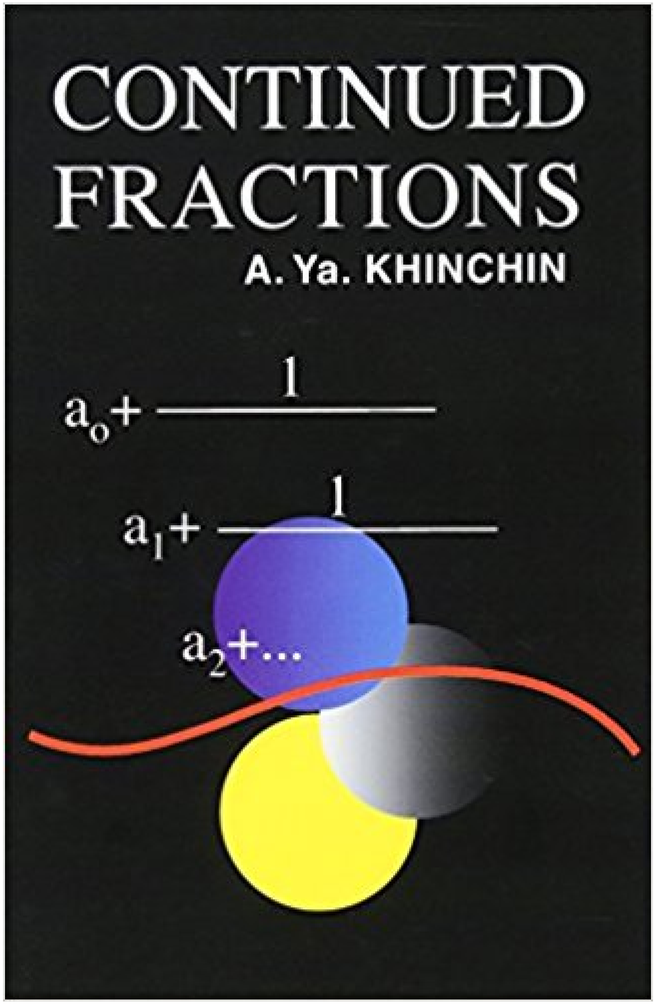
Continued Fractions (Dover Books On Mathematics)



Synopsis

In this elementary-level text, eminent Soviet mathematician A. Ya. Khinchin offers a superb introduction to the positive-integral elements of the theory of continued functions, a special algorithm that is one of the most important tools in analysis, probability theory, mechanics, and, especially, number theory.Presented in a clear, straightforward manner, the book comprises three major chapters: the properties of the apparatus, the representation of numbers by continued fractions and the measure theory of continued fractions. The last chapter is somewhat more advanced and deals with the metric, or probability, theory of continued fractions, an important field developed almost entirely by Soviet mathematicians, including Khinchin.The present volume reprints an English translation of the third Russian edition published in 1961. It is not only an excellent introduction to the study of continued fractions, but a stimulating consideration of the profound and interesting problems of the measure theory of numbers.

Book Information

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Customer Reviews

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Continued fractions are fractions with multiple denominators; e.g., the golden ratio = 1+1/(1+1/(1+..., the square root of 2 = 1+1/(2+1/(2+.... Indeed, all quadratic irrationals have repeating continued fractions, giving them a convenient and easily memorable algorithm. Continued fractions may be truncated at any point to give the best rational approximation. For example 1/pi = 113/355 -- something that is very easy to remember (note the doubles of the odd numbers up to five). Therefore, an excellent approximation for pi becomes 355/113. The fraction approximates pi to an error better than 3E-7, more than accurate enough for any practical use including astronomy. Thus for both transcendental and analytical irrationals, continued fractions are enormously useful.Never heard of them? You’re not alone. The first recorded instance of continued fractions was by Lord Brouncker in the 17th century which makes them a relatively new addition to mathematics. Nor are they taught in typical undergraduate scientific curricula. Notwithstanding, if they were discovered by the Pythagoreans, history may have been much different.The Pythagoreans were a mystical sect that believed that all things geometric could be described by rational numbers (i.e., wholes and fractions). Something like the square root of two was clearly geometric (the diagonal of the unit square) yet, irrational. Legend has it that Hippasus (5th century B.C.) was expelled from (or killed by) the Pythagorean school for proving the irrationality of a number such as the square root of 2 or the golden ratio. This ultimately destroyed the Pythagorean religion. Had the theory of continued fractions been discovered at this time, irrationals would have been reduced to infinite fractions of whole numbers and the religion may have well survived until (or perhaps interfered with) the advent of Christianity.This monograph by the Russian mathematician, Aleksandr Khinchin, is a very inexpensive way to obtain a good introduction. The author died in 1959, however, his third edition of the book was translated into English in 1964 and revised in 1997. The monograph is less than 100 pages and organized into three chapters: I. Properties of the Apparatus; II. The Representation of Numbers by Continued Fractions; III. The Measure Theory of Continued Fractions. The book also has a brief and inadequate index.Some of the fascinating things one will learn is that if a/b < c/d then the value (a+c)/(b+d) is always intermediate: a/b < (a+c)/(b+d) < c/d. Repeated application of this algorithm gives an infinitely divisible and ordered sequence of rational numbers; e.g., the infinite sequence 1/1, 1/2, 1/3, 1/4... 0/1 is one such application of the theorem. One can also prove that 355/113 is the best three digit rational approximation to pi -- a result of remarkable accuracy. One will also learn that all rational numbers can be represented by finite continued fractions. For example, 54/17 = 3+1/(5+1/(1+1/2)). Therefore, continued fractions are capable of representing all real numbers: some as finite fractions (e.g. 54/17 or any rational), some as non-terminating repeating fractions (e.g., square root of 2 or any quadratic root), some as non-repeating non-terminating fractions having a pattern [e.g., Euler’s constant, e = 2+1/(1+1/(2+1/(1+1/(1+1/(4+1/(1+1/(1+1/(6+....], and others as non-terminating non-repeating fractions without pattern [e.g., pi = 3 + 1/(7+1/(15+1/(1+1/(292+1/(1+1/(1+...]. Regarding the latter, if one can derive an infinite series representation, then it is possible to recast the regular continuing fraction (numerator of 1s) as an irregular continuing fraction having a pattern [e.g., pi = 3+1/(6+9/(6+25/(6+49/(6+...].Thus, continued fractions are a powerful mathematical device, and this book provides a reasonable if not brief introduction.

Covers some ideas using Continued Fractions. It doesn't really go into depth, but it is a pretty thin book. I bought this book a few weeks ago to brush up on some of my unused math learning, and to learn some more about continued fractions. I know they can be used to expand expressions and to define stuff like the value of pi, but the author didn't really cover stuff like that. Rather than having examples, he went into the more general cases. I don't have that much experience in that kind of thing, my math stops short at the end of Calculus II, but it was pretty informative.

Short, readable book, wealth of applications, this little gem provides a doorway into deep number theory. Gives insight into transcendence theory, as well as bicycle gearing -- Huygen's problem. There are some minor typographic errors scattered about the text which are annoying, especially for such a typographic challenge as continued fractions. For the price, you can't go wrong.

This is a great book by a master. You need to have taken a course in analysis, and have some idea about measure theory, to appreciate it.

The first two chapters are an excellent introduction for the subject. Though the book lacks examples and exercises, the first chapter is very well explained and organised letting us (the begginers) to grasp the main concepts of what continued fractions are.From the middle of chapter two onwards, it gets way more convoluted and the proofs much harder to understand.For the book price it is a very recommendable purchase.

Great text on the theory and application of continued functions.

A wonderfully written, clear exposition of advanced material which, however, begins simply enough to lure one in.

This material in this classic work is mostly accessible to the undergraduate level engineer, mathematics, or other reader with similar background, but a good portion of this book is about measure theory of continued fractions, which is more easily accessed by those who have some mathematics background at the graduate level.Measure theory is the theory of the fraction of the extent of a domain that is mapped from the range of inputs to a function. Measure theory is used primarily by Khinchin and his students, but similar work is posed in terms of probability theory and other contexts by other mathematicians. Once this is understood (and it is hinted at by a footnote or two from the translator), this material becomes as accessible as the rest of the material.The book begins with a minor aside in a proof of convergence of continued fractions that have real partial numerators and denominators, whose partial numerators are all unity, and the sum of whose partial denominators diverges. Since the simple classical number-theoretic continued fractions are the subject of the book, this proof clearly includes all such continued fractions. This minor excursion from number theory and algebra is a significant advantage to this particular book as it provides a bedrock for later rate-of-convergence discussions.The related field of analytic theory of continued fractions that was explored by Riemann, Stieltjes, Tchebychev, PadÃƒÂ©, Hamburger, CesÃƒÂ ro, and others that are contemporary to Khinchin (memorable classic by H.S. Wall was published in 1948, long after this book was written), is not ignored entirely.

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